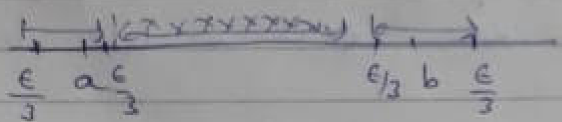
13/05/2020 (21)

Ex of a Hausdorff space :– Let $a, b \in R$ be arbitrary s.t.

$$|a-b| = \epsilon > 0 \quad \text{and} \quad a > b$$

Let

$$G = \left(a - \frac{\epsilon}{3},\ a + \frac{\epsilon}{3}\right),\ H = \left(b - \frac{\epsilon}{3},\ b + \frac{\epsilon}{3}\right)$$



$G \cap H = \phi \qquad G, H \in \mathfrak{I}$

proves that $(R, \mathfrak{I})$ is a Hausdorff space.

Ex Show that every discrete space is a H. Space.

Proof :– Let $(X, \mathfrak{I})$ be a topological space and $x, y \in X$ s.t. $x \neq y$. From the def of discrete space, $\{x\}$ and $\{y\}$ are $\mathfrak{I}$– open sets.

obviously $\{x\} \cap \{y\} = \phi$, $x \in \{x\}$, $y \in \{y\}$.

proves that $(X, \mathfrak{I})$ is a H. Space.

Thm :– Show that every Hausdorff space is a T_1–space. Is the converse true ?

Proof) – Suppose $(X, \mathfrak{I})$ is a T_2– space.

To prove that $(X, \mathfrak{I})$ is a T_1– Space

By def. of T_2– Space we have for $x, y \in X$ s.t $x \neq y$ we can find disjoint sets $G, H \in \mathfrak{I}$ s.t $x \in G$, $y \in H$

$$G \cap H = \phi,\ x \in G,\ y \in H \Rightarrow x \in G, y \notin G;\ y \in H, x \notin H$$

Hence for $x, y \in X$ st. $x \neq y$ $\exists$ $G, H \in \mathfrak{I}$ s.t $x \in G$, $y \notin G$; $y \in H$, $x \notin H$

Proves that $(X, \mathfrak{I})$ is a T_1– Space.

Converse ; – we have to take an example to prove it

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Problem :- Show that an indiscrete space consisting of at least two points is not a Hausdorff space.

Sol :- Let $\mathfrak{J}$ be an indiscrete topology on a space X consisting of at least two points. Then by def $\mathfrak{J} = \{\phi, X\}$

This proves that $\exists$ no pair of non-empty disjoint open sets, $(X,\mathfrak{J})$ is not H- space.

Problem :- Define T_1- space and show that T_1-space is T_0- space.

Proof :- Def. do urself.

second part :- Given $(X,\mathfrak{J})$ is a T_1- space.

To prove that X is T_0- space.

given, $x, y \in X$ $\exists$ $G, H \in \mathfrak{J}$ st

$$x \in G, y \notin G \text{ and } y \in H, x \notin H$$

from this it follows that

$x \in G$, $y \notin G$

proves that $(X,\mathfrak{J})$ is a T_0- space.